(7 pages) **Reg. No. :** ....

Code No. : 6842 Sub. Code : PMAE 22

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics

ELECTIVE - DISCRETE MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. A compound proposition that is always true no matter what the truth values of the propositions that occur in it is called ___________.

   (a) logic (b) tautology

   (c) inverse (d) truth value

2. The conditional statement $p \to q$ is false when p is true and q is ___________.

   (a) true (b) false

   (c) both (d) none

3. The number of different bit strings of length seven are ___________.

(a) 7!   (b) 7–1!

(c) 2^7 (d) 2^6

4. By product rule the number of different subsets of a finite set S is ___________.

(a) $|S|$ (b) $2^{|S|}$

(c) $2^{|S|-1}$ (d) $|S|-1$

5. A relation R on a set A is called ___________ if $(b, a) \in R$ whenever $(a, b) \in R$

(a) refluxive   (b) symmetric

(c) transitive   (d) antisymmetric

6. A domain of n-ary relation is called a ___________ when the value of the n-tuple from this domain determines the n-tuple.

(a) tables   (b) primary key

(c) extension   (d) inclusion

7. The value of 1 + 1 = ___________ in Boolean function.

(a) 1   (b) 0

(c) 2   (d) –1

8. A ____________ is a Boolean variable or its complement.

(a) inverse (b) literal

(c) minterm (d) none

9. The ____________ which accepts the value of one Boolean variable as input and produces the complement of this value as its output

(a) inverter (b) gate

(c) nor (d) and

10. Cells are said to be ____________ if the minterms that they represent differ in exactly in one literal.

(a) normal (b) complement

(c) adjacent (d) none

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write the truth table for the biconditional $p \leftrightarrow q$.

Or

(b) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

12. (a) How many one to one functions are there from a set with m elements to one with n elements?

Or

(b) How many cards must be selected from a standard deck of 52 cards to guarantee that atleast three cards of the same suit are chosen?

13. (a) How many refluxive relations are there on a set with n elements?

Or

(b) Define $M_{R_1 \cup R_2}$ and $M_{R_1 \cap R_2}$. Suppose that the relations R_1 and R_2 on a set A are represented by the matrices $M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ and $M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ then what are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$.

14. (a) Find the sum of products expansion for the function $F(x, y, z) = (x + y)\bar{z}$.

Or

(b) Write down the Boolean identities with their names.

15. (a) A committee of 3 individuals decides issues for an organization. Each individual votes either year or no for each proposal. A proposal is passed if it received atleast two yes votes. Design a circuit that determines whether a proposal passes.

Or

(b) Find K-maps for (i) $xy + \bar{x}y$ (ii) $x\bar{y} + \bar{x}y$ (iii) $x\bar{y} + \bar{x}y + \bar{x}\,\bar{y}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that $\forall x(P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ are logically equivalent.

Or

(b) (i) Write down the truth table for $(p \vee \neg q) \rightarrow (p \wedge q)$.

(ii) Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

17. (a) Each user on a computer has a password which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain atleast one digit. How many possible passwords are there?

Or

(b) Prove that every sequence of n^2+1 distinct real numbers contains a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing.

18. (a) Find the zero one matrix of the transitive closure of the relation R where $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

Or

(b) Prove that the relation R on a set A is transitive iff $R^n \subseteq R$, $n = 1, 2, \ldots$

19. (a) Translate the distributive law $x + yz = (x+y)(x+z)$ into a logical equivalence.

Or

(b) Translate $1.0 + \overline{0+1} = 0$ into logical equivalence.

20. (a) Use K-maps to minimize these sum of products expansions.

(i) $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\,\bar{y}\bar{z}$

(ii) $x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\,\bar{y}z + \bar{x}\,\bar{y}\bar{z}$

Or

(b) Construct circuit that produce the following outputs :

(i) $(x+y)\overline{x}$

(ii) $\overline{x}\left(\overline{y+\overline{z}}\right)$

(iii) $(x+y+z)(\overline{x}\,\overline{y}\,\overline{z})$.